Math 311W, Section 2 : Concepts of Discrete Mathematics
Introduction to the mathematical activity

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Spring 2023

Abstract

I wrote this short reflection about the mathematical activity for students following the Math 311W course "Concepts of Discrete Mathematics" at PSU.

What is mathematics ?

An evolving organism. Mathematics, as an activity practised by our human minds living in our society nowadays, consists of several processes which interact and influence one another in various ways. One may thus compare mathematics to an organism evolving in time. Its definition cannot be dissociated from its historical evolution, but we may still give a rough description of these processes and how they interact at this point in time.

1) Experimental acquisition of the logical patterns governing the universe. By observing the occurrence of events surrounding us, we develop an intuition of logic, that is a system of inference rules predicting the outcome of an experiment placed in some initial state, or the result of an algorithm on some inputs. This reasoning is unconscious at first and refined by experience (including observation and debate, or learning and teaching). The conscious exploration of this logical system is part of the mathematical activity, close to philosophy and computing sciences.

2) Modeling natural phenomena and predicting their behaviour. With these logical rules in mind, we return to the observation of natural phenomena or structures (like chemical reactions, propagation of light, motions of bodies), and extract simple models by abstracting the laws which describe them best (the axioms for the observable patterns). We ask questions about the models and explore their outcomes, to predict and compare with the behaviour of the natural structures from which they arose. This experimental part of mathematics is close to the so called natural sciences, from the standard model of particle physics to the biological theory of evolution.

3) Constructing formal structures and studying them To analyse the models we put them into a rigorous framework by making precise definitions, developing problem solving strategies, and tactics of proofs. We invent new structures to simplify, abstract or generalise the models we have already extracted, and use them to investigate new patterns observed in nature. Their study give rise to various theories (such as graph theory, euclidean geometry, number theory, measure theory,...). The construction of axiomatic theories interacts with linguistics.

In summary: We acquire from observation the logical patterns governing our universe, use it to predict natural phenomena by studying simplified models, which in turn become the subject of investigation and the source of problem solving.

After some time, the last step bootstraps the first, as we develop a specific language (or refine the existing one) to manipulate and communicate the logical rules inferred from our initial experience. Indeed, this thinking process as well as the learning and teaching activities which result in this communication add up to our experience and contribute to the evolution of mathematics.

Remark (Mathematics and literature)**.** *One may compare the investigation of problem solving strategies and the techniques for devising proofs to the exploration of literary devices and the confection of stylistic poems or puns. Yet, in the same way as literature and humour arises from a context and that one cannot dissociate form and content, we must not forget that mathematics is not reduced to those formal activities and that its meaning is deeply rooted in the real world.*

Remark (From probabilistic events to deterministic propositions)**.** *We do not have a complete knowledge of the events happening in nature (for instance their microscopical properties may not be accessible to our observations), so our observations and predictions only hold with some degree of certainty, and this leads to a probabilistic calculus of events.*

An idealisation of those events consists in replacing them with binary variables (for instance the result to a questions concerning their macroscopic state), and we are led to the so called propositional calculus of Boolean variables.

One may thus view the calculus of binary propositions as a shadow, an approximation, a limit, an idealisation or a combinatorial invariant of the calculus of probalistic events.

In this course

In this course we will introduce to a formal language enabling a rigorous manipulation of our intuitive logic, define and study elementary structures, develop some problem solving strategies and techniques of proof to investigate them.

Some references.

For those in search of pedagogical essays related to the course, let us mention the *Philosophy and Fun of Algebra* [Boo09] by Mary Boole, and suggest *Playing with Infinity* [P76́] by Rosza Péters.

You may also find amusement in the hybrid works of Lewis Carroll and Raymond Smullyan [Car86, Smu82], or learn about first order logic in [Car96, Smu14] (taste the former with a pinch of salt, and devour the latter).

If you wish to pursue scientific studies, I recommend you read in a near future the essays on *mathematics and plausible reasoning* by George Polya. The first volume [Pol54a] illustrates the role of *induction and analogy in mathematics*, while explaining many mathematical principles and problem solving strategies. The second volume [Pol54b] concerns the *patterns of plausible inference*, tying probability with logic and psychological behaviour.

Many great minds, have expressed their thoughts concerning the relations between mathematics, physics and language. Let us mention from the last century, the names of Gian-Carlo Rota, Yuri Manin, René Thom, Misha Gromov. The following references lie way beyond the scope of the course, but you may appreciate to keep them in your anti-library : [Man07, Tho72].

References

[Boo09] Marie Everest Boole. *Philosophy and Fun of Algebra*. 1909. Online archive, Online audio.

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[Man07] Yuri I. Manin. *Mathematics as metaphor*. American Mathematical Society, Providence, RI, 2007. Selected essays of Yuri I. Manin, With a foreword by Freeman J. Dyson.

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[Pol54a] G. Polya. *Induction and analogy in mathematics. Mathematics and plausible reasoning, vol. I.* Princeton University Press, Princeton, N. J., 1954.

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[Smu82] Raymond Smullyan. *The Lady or the Tiger ? & Other Magical Puzzles*. Knopf Doubleday, 1982. author website.

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[Tho72] René Thom. *Stabilité structurelle et morphogénèse*. Mathematical Physics Monograph Series. W. A. Benjamin, Inc., Reading, Mass., 1972. Essai d'une théorie générale des modèles.