Mathematical thinking

We collected from [MBS10] and [Pol54a, Pol54a] some guiding principles to mathematical thinking.

Curiosity is the main drive for research and progress. It consists in observing our surroundings, questioning the phenomena, and feeling the desire to understand them.

We must not take the simple for granted, that is often where lies the essence of understanding. We should also examine the banal carefully, as it is the source of biased and hidden assumptions (to think out of the box we must recognise its boundaries).

Progress arises from a back and forth between an attitude of confidence (trusting our knowledge and intuition), and an attitude of doubt (questioning assumptions, reasoning, conclusions).

Thought processes

Specializing consists in choosing examples:

randomly	to get a feeling for the behaviour and find a pattern;
systematically	to test a pattern and prepare ground for generalisation;
artfully	to test a generalisation, construct a counter-example.

Conjecturing what seems likely to be true, by finding

patterns	arising from the experiments;
analogies	with previously solved problems.

Justifying why it is true, going back and forth between an attitude of

confidence	during which you sketch an argument and write a proof;
skepticism	during which you question the hidden assumptions and add the details.

Generalising the ideas to broader and other situations, can be achieved by:

delimiting	which hypotheses were needed, and what conclusion was actually attained;
varying	the hypotheses or the conclusion, for instance by weakening or strengthening them;
imitating	the shape of an argument in analogous contexts.

Phases of work

Entry Take your time to get familiar with the problem, .

Target	What do i want ? What am i asked to show ?
Source	What do i know ? Select the relevant information.
Steps	What intermediate knowledge would help as stepping stones ?

Attack Most of the time we are stuck, in search for a grasp and a place to start.

Stuck	Experiment, compute, introduce an object.
Aha!	Recognise a pattern and formulate a conjecture.
Write	Devise an argument, write a proof.

Review Take your time to learn from your work, that is where you make progress.

Check	computations, arguments, that the resolution answers the question
Reflect	on the key ideas, about simplifying the argument, by investigating corollaries.
Extend	to a wider context, by finding a new solution, by varying constraints.